Generalizing Solow's Growth Model

Author : James Annable

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Providing the mainstream literature's go-to explanation for trend macrodynamics is Robert Solow's (1956, 1957) neoclassical growth theory. Along with Sir Arthur Lewis's two-venue theory featured last week, Solow broke the problematic mechanical link between saving and growth.

Basic model. Recall last week's blog. The Lewis two-sector theory, comprising a large-establishment venue (LEV) and a small-establishment venue (SEV), generates growth in labor productivity via accumulating capital and the transfer of workers to more productive jobs. Solow's contribution is the introduction of technical change, which Lewis would locate in LEV firms. In its Harrod-neutral formulation, the Solow model is: $X(t)=f(K(t), A(t)H(t))$, where X denotes real output, K is capital, H is labor hours, and A represents technology. (In their lowercase versions, the variables represent rates of change.) Solow (2000, p.103) later expanded the definition of A to include "… worker skills and attitudes toward work, managerial and administrative habits, interpersonal attitudes, social norms and institutions, and no doubt many other hard and soft characteristics of the economic and social environment."

Assuming constant returns to scale and diminishing returns, the Solow model's dynamic path can be derived: $x(t) \cong a(t)-S_K(t)k(t)-S_H(t)h(t)$, and $a(t) \cong x(t)-S_K(t)k(t)-S_H(t)h(t)$, where a is the rate of change of the shift factor typically interpreted as technical change, S_K is the share of total income paid to capital, and S_H is labor's income share. Versions of the Solow model have been widely used in estimation exercises, which were themselves a growth industry for an earlier generation of macroeconomists.

The neoclassical growth theory was recognized almost immediately as an important advance in macrodynamic thinking, replacing "knife-edge" growth paths with more robust steady-state paths motivated by capital accumulation and technological change. The approach showcased the longstanding liberal economic agenda of free markets, free trade, and sound money. Solow's 1987 Nobel-Prize citation emphasized his provision of "a framework within which modern macroeconomic theory can be structured", and his model continues to generate broad, understandable appeal as a guide to aggregate analysis.

The simple model, however, notably stumbles when asked to account for some really important phenomena, including the mid-19th century growth acceleration often referred to as the Great Fact. (See website's e-book, chapter 1.) Consequential limitations of the Solow model result from its construction within a market-centric framework. Coherent single-venue general-equilibrium (SVGE) modeling provides room neither for the increasing returns to scale nor two-venue labor transfer that critically combined to motivate the jump in global living-standards growth that began a century and a half ago. The Project's two-venue general-equilibrium modeling, by contrast, provides the heterogeneous marketplace and workplace venues of rational price-mediated exchange needed to accommodate increasing returns and inter-sectoral labor transfer.

GEM version. A strength of the Solow framework is its remarkable versatility, enabling its relevance well beyond mainstream market-centric analysis. Most critically, substituting $\acute{Z}=E/H$ into the core equation, more explicitly motivating the "effectiveness of labor" and the role of pure profit, makes Solow production consistent with the GEM Project's cyclical and trend modeling: $X(t)= f(K(t),A(t)Z(t)H(t))$. A is now understood to reflect the technical efficiency of labor, once the influence of capital-labor intensity is held constant. (Recall the post two weeks ago.) The product of A and Z measures general worker productivity and is the vehicle through which optimizing employee-employer workplace behavior introduces into the Solow framework the meaningful wage rigidities (MWR) that combine with adverse nominal demand disturbances to induce periodic involuntary job and income loss. In empirical exercises, that rational macro instability is captured by the Solow residual. (See next week's post.)

The MWR nominal-to-real channel along with capital investment, technological change, scale economies, and inter-venue labor transfer (the last three are also reflected in estimation residuals) enables intuitive, policy-relevant modeling of instability and trend growth. Properly generalizing labor input ($H=E/Z$) endows Solow's iconic model with analytic range well beyond his original aspirations (2001, p.19): "… it was clear from the very beginning what I thought [the neoclassical growth model] did not apply to, namely short-run fluctuations in aggregate output and employment, what used to be called the business cycle…. In those days I thought growth theory was about the supply side of the economy, whereas the business cycle was mostly to be analyzed in

terms of changes in aggregate demand."

The generalized-exchange version of the Solow theory is obviously important, incorporating business cycles induced by nominal demand disturbances as well as nonstationary labor-productivity macrodynamics. As hoped by RBC theorists, it provides a coherent analytic framework for understanding a broad range of important events, including the late-19th century upward shift in living-standards growth associated with the Second Industrial Revolution, the 1930s depression, the 1970s stagflation, the extraordinary growth in central and east Asia beginning toward the end of the 20th century, and the propagation of financial crises that have characterized the early 21st century. (See the e-book, chapter 10.)

Putting the analytic strands together, Solow's focus on technical change provides one of the most important insights in the history of macroeconomics. Beginning in the 19th century, the broad on-going reorganization of the global economy generated the 1% or so annual multifactor productivity growth that broke the world out of millennia of living-standard stagnation. Those beneficial macrodynamics are centrally informed by Lewis inter-venue labor transfer. The generalized-exchange interpretation of Solow growth also captures the transformation of the production landscape, reflected in the advent of large bureaucratic corporations, that created the rational MWR nominal-to-real channel. Macroeconomics was henceforth tasked with making sense out of variable periods of instability (rooted in aggregate demand and producing substantial welfare loss) as well as time-varying trend growth (rooted in aggregate supply and producing substantial welfare gain). Market-centric theorists, reflecting their inability to coherently suppress wage recontracting and derive involuntary job loss, have been unsurprisingly reluctant to accept the GEM Project's intuitive explanation of the Solow residual. (See next week.)

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